DELFT UNIVERSITY OF TECHNOLOGY
Faculty of Electrical Engineering, Mathematics
and Computer Science
Mediamatics Department

Computer Graphics (in2905-I)
2 November 2007, 09.00 - 12.00 h.

correct answers with elaborate explanation

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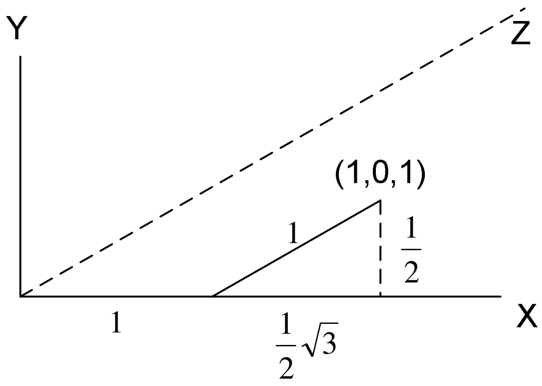
answer 1

The angle between the projectors and the projection plane is $\alpha = 45^\circ$
The length of the projection of a line, which is perpendicular to the projection plane, relative to its real length is $\frac{1}{\tan\alpha} = 1$



In the image above you can see that (1, 0, 1) in 3D space is mapped on $\left(1+\frac{1}{2}\sqrt{3}, \frac{1}{2}\right)$ in 2D space:

$$\begin{pmatrix}1\\0\\1\\1\end{pmatrix} \rightarrow \begin{pmatrix}1+\frac{1}{2}\sqrt{3}\\ \frac{1}{2}\\ 0\\ 1\end{pmatrix}$$

. From the given parallel projection matrix we find:

$$\begin{pmatrix}1 & 0 & p & 0\\ 0 & 1 & q & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\begin{pmatrix}1\\0\\1\\1\end{pmatrix} = \begin{pmatrix}1+p\\ q\\ 0\\ 1\end{pmatrix}$$

So

$$\begin{pmatrix}1+p\\ q\\ 0\\ 1\end{pmatrix} = \begin{pmatrix}1+\frac{1}{2}\sqrt{3}\\ \frac{1}{2}\\ 0\\ 1\end{pmatrix}$$

and therefore

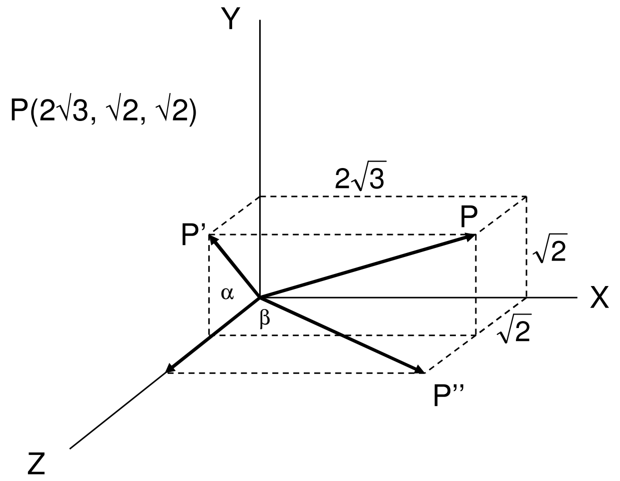
$$p = \frac{1}{2}\sqrt{3}$$
$$q = \frac{1}{2}$$

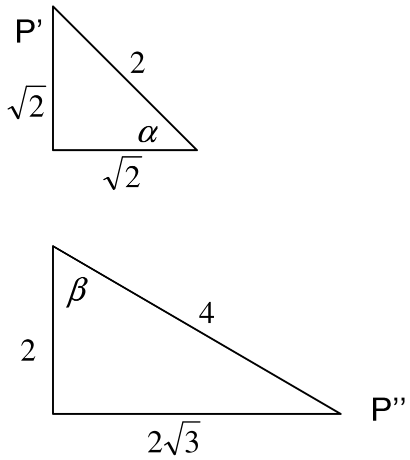
Correct answer A

answer 2

Idle events are generated if the computer has no other events to handle. It was used for the digital clock in the program for the first lab assignment of this course. See also the syllabus (section 1.4.1).
Correct answer C

answer 3





From the two rectangular triangles in the figure we find

$\sin\alpha = \frac{\sqrt{2}}{2} = \frac{1}{2}\sqrt{2}$ and $\cos\alpha = \frac{\sqrt{2}}{2} = \frac{1}{2}\sqrt{2}$

$\sin\beta = \frac{2\sqrt{3}}{4} = \frac{1}{2}\sqrt{3}$ and $\cos\beta = \frac{2}{4} = \frac{1}{2}$ (the z-coordinate of P'' is 2 because the length of P' is 2)

So $\alpha = 45^\circ$ and $\beta = 60^\circ$. However, the angle β is in clockwise direction when looking from the positive Y-axis towards the origin, so $\beta = -60^\circ$.
Correct answer D

answer 4

An x-direction shearing transformation relative to y=0 can be represented by the matrix $\begin{pmatrix} 1 & Shx & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. An x-direction shearing transformation relative to y=2 can be built from three transformations: 1. translation with vector $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$; 2. x-direction shearing relative to y=0; 3 translation with vector $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$. The matrix for this composite transformation is

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & Shx & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & Shx & -2Shx \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Because P (4, 4) is mapped on P' (10, 4) we find:

$$\begin{pmatrix} 1 & Shx & -2Shx \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 4 \\ 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 10 \\ 4 \\ 1 \end{pmatrix}.$$

$4 + 4Shx - 2Shx = 10$

$2Shx = 6$

$Shx = 3$

Correct answer C

answer 5

Two polygons are in parallel planes if and only if the normal vectors are dependent, i.e. one normal vector is a scalar times the other normal vector.
We determine the normal vectors of the three planes:

ABC: $(\bar{b} - \bar{a}) \times (\bar{c} - \bar{a}) = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} \times \begin{pmatrix} 4 \\ -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \\ -8 \end{pmatrix} \approx \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix}$

DEF: $(\bar{e} - \bar{d}) \times (\bar{f} - \bar{d}) = \begin{pmatrix} 3 \\ -6 \\ -3 \end{pmatrix} \times \begin{pmatrix} -1 \\ -2 \\ -2 \end{pmatrix} = \begin{pmatrix} 6 \\ 9 \\ -12 \end{pmatrix} \approx \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}$

GHI: $(\bar{h} - \bar{g}) \times (\bar{i} - \bar{g}) = \begin{pmatrix} -2 \\ -1 \\ -2 \end{pmatrix} \times \begin{pmatrix} -4 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \\ -8 \end{pmatrix} \approx \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix}$

Only ABC and GHI have dependent normal vectors.

Correct answer B

answer 6

statement (I)
With the 4-connected boundary fill algorithm, the polygon in the left figure cannot be filled correctly because the algorithm will stop filling after the first pixel. All other figures can be filled correctly with this algorithm. So, statement (I) is correct.

statement (II)
The polygon in the right figure can be filled correctly with the 8-connected flood fill algorithm. When used for filling the other three polygons, the algorithm will “escape” through the mazes of the boundary and the area outside the polygon will also be filled. So, only the rightmost figure can be filled correctly with this algorithm. So, statement (II) is correct.

Correct answer A

answer 7

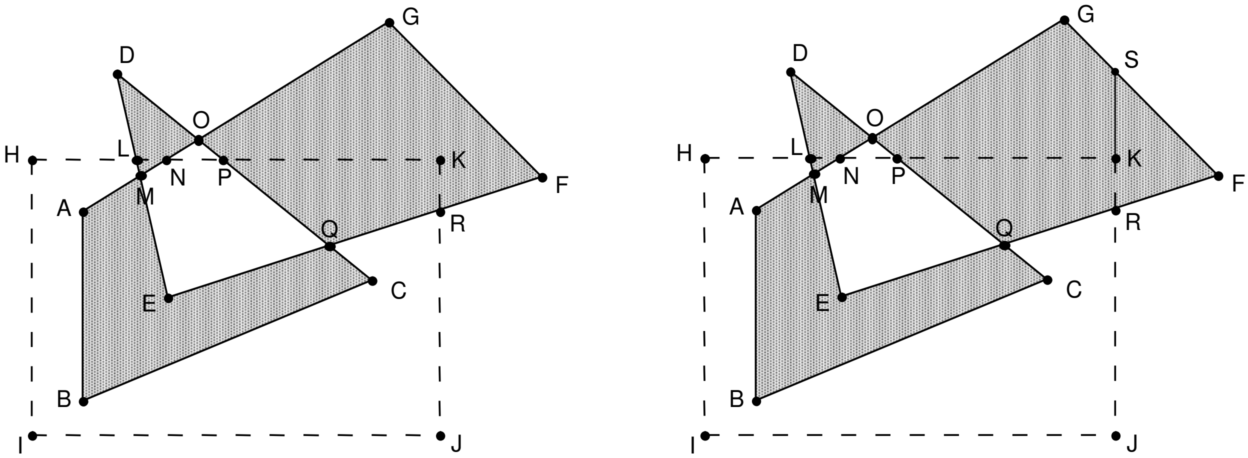
statement (I):

A translation over a vector $\begin{pmatrix} tx \\ ty \end{pmatrix}$ can be written as a vector addition $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} tx \\ tx \end{pmatrix}$. It cannot be represented with a 2 x 2 matrix. However, in homogeneous coordinates it can be written as $\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$. In a similar way a 3D translation can be represented with a 4 x 4 matrix in homogeneous coordinates. So, statement (I) is correct.

statement (II):
A perspective transformation with COP = (0, 0, 0) and view plane $z = -d$ can be written as $\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1/d & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$. After division of x', y' and z' by the homogeneous coordinate w' we have found the coordinates of the projected point. Again, we cannot use a 3 x 3 matrix. So, statement (II) is correct.

Correct answer A

answer 8



The final result after clipping is independent of the order of clipping against the 4 window boundaries. Let us use the order: left, right, bottom, top. After every clipping step, the input polygon has been converted into an output polygon. Then, the output polygon is the input polygon for the next step.
Clip against the left window boundary: ABCDEFG --> ABCDEFG.
Clip against the right window boundary: ABCDEFG --> ABCDERSG
Clip against the bottom window boundary: ABCDERSG --> ABCDERSG
Clip against the top window boundary: ABCDERSG --> ABCPLERKN

Correct answer A

answer 9

If a model consists of only one object and the object is convex, then all front faces of the model are totally visible in the final image and all back faces are totally invisible. After back face removal only the front faces remain and so the hidden surface removal problem has been solved.

Correct answer B

answer 10

P' is the intersection point of the projector through P = (3, 6, -3), i.e. $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \lambda \begin{pmatrix} 3 \\ 6 \\ -3 \end{pmatrix}$ and the view plane z = -10. This is the point with $-10 = -3\lambda$, so $\lambda = \frac{10}{3}$. Substituting this value of λ in the parameter representation of the projector gives $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{10}{3}\begin{pmatrix} 3 \\ 6 \\ -3 \end{pmatrix} = \begin{pmatrix} 10 \\ 20 \\ -10 \end{pmatrix}$. So P' = (10, 20, -10).

<u>Alternative (if you know the matrix for a perspective projection by heart):</u>

The matrix for the perspective projection is $M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1/d & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1/10 & 0 \end{pmatrix}$.

$$p' = M \cdot \begin{pmatrix} 3 \\ 6 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \\ -3 \\ 3/10 \end{pmatrix} \cong \begin{pmatrix} 10/3 \cdot 3 \\ 10/3 \cdot 6 \\ 10/3 \cdot -3 \\ 10/3 \cdot 3/10 \end{pmatrix} = \begin{pmatrix} 10 \\ 20 \\ -10 \\ 1 \end{pmatrix}$$. So P' = (10, 20, -10).

Correct answer A

answer 11

The *cross product (Dutch: uitwendig product)* of two independent vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ is

- a vector perpendicular to $\mathbf{v}_1$ and $\mathbf{v}_2$
- with direction determined by the right hand rule
- and length equal to the area of the parallelogram spanned by the two vectors

The *dot product (Dutch: inwendig product)* of two independent vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ is the product of the length of $\mathbf{v}_1$ and the length of the projection of $\mathbf{v}_2$ on $\mathbf{v}_1$.

Correct answer D

answer 12

In the z-buffer z-values (depth values) are stored of the polygons which are per pixel closest to the observer during executing the z-buffer algorithm. So repeatedly, for x and y of a pixel we calculate, using a plane equation, the z-coordinate for a polygon p, whose projection overlaps pixel (x, y). If the z-coordinate is smaller than the z-value in the depth buffer, then the z-value in the depth buffer is substituted by the new z-value (closer to the observer).
Correct answer D

answer 13

The easiest way to find out what is the correct parameter representation of the line is by using the following property: points on the line about which to rotate do not change position by performing the rotation. So $P = (x_p, y_p, z_p)$ is on the line if and only if $p = M \cdot p$ with $p = \begin{pmatrix} x_p \\ y_p \\ z_p \\ 1 \end{pmatrix}$.

For all four lines take the point with $\lambda = 1$. One of these four points will not change position by performing the transformation with matrix *M*. As we can see below, this is the point on the line in b

a) $M \cdot \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \\ 1 \end{pmatrix}$ b) $M \cdot \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix}$ c) $M \cdot \begin{pmatrix} 1 \\ -1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix}$ d) $M \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ -1 \\ 1 \end{pmatrix}$

Correct answer B

answer 14

$$\begin{pmatrix} -\frac{250}{3} & -\frac{200}{3} & 150 \\ 0 & -200 & 200 \\ \frac{1}{3} & -\frac{7}{12} & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{250}{3} \\ 0 \\ \frac{5}{12} \end{pmatrix} \cong \begin{pmatrix} \frac{12}{5} \cdot \frac{250}{3} \\ \frac{12}{5} \cdot 0 \\ \frac{12}{5} \cdot \frac{5}{12} \end{pmatrix} = \begin{pmatrix} 200 \\ 0 \\ 1 \end{pmatrix}$$

. So D = (0,1) is mapped on Q = (200,0).

Correct answer B

answer 15

ABC is mapped on A'B'C'. So $\begin{pmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 3 & 1 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 3 & 4 \\ 1 & 3 & 1 \\ 1 & 1 & 1 \end{pmatrix}$. From this matrix equation we find:

$$\begin{array}{l} a + b + c = 3 \\ 3a + b + c = 3 \\ a + 2b + c = 4 \end{array} \qquad 2a = 0 \qquad a = 0$$

By substituting this value of a we find: $\begin{array}{l} b+c=3 \\ 2b+c=4 \end{array}$ $b=1$ $c=3-1=2$

Correct answer C

answer 16

glOrtho must be called with 6 arguments.
In OpenGL 2D graphics is 3D graphics in the plane z = 0.
glOrtho defines the view volume in 3D. The last two arguments define the (z-coordinates of the) near plane and the far plane. The plane z = 0 must be situated between the near plane and the far plane. So, -1 and 1 is a proper choice for the z-coordinates of the near plane and the far plane.
Correct answer D

answer 17

Blocks of 2 x 2 sub-pixels are used to find a color for a pixel. Intensities for sub-pixels are 0 (black) or 1 (white). In a 2 x 2 block of sub-pixels the sum of the sub-pixel intensities is 0 (all sub-pixels are black), 1, 2, 3 or 4 (all sub-pixels are white). So, there are 5 possible intensity levels for a pixel.
Correct answer C

answer 18

$M_1 = T(-1,0)\cdot S(1,-1)\cdot T(1,0) = S(1,-1) = Sp_{x-axis}$ is a reflection in the x-axis

$M_2 = T(1,0)\cdot S(1,-1)\cdot T(-1,0) = S(1,-1) = Sp_{x-axis}$ is also a reflection in the x-axis

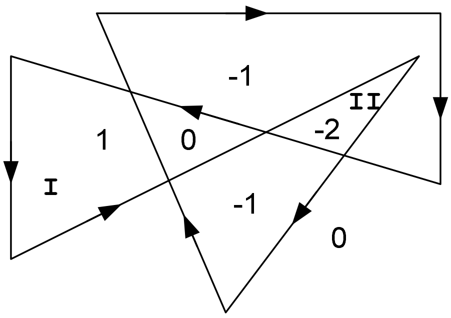
$M_3 = R(45^\circ)\cdot S(1,-1)\cdot R(-45^\circ) = Sp_{y=x}$ is a reflection in the line y=x

$M_4 = R(-45^\circ)\cdot S(-1,1)\cdot R(45^\circ) = Sp_{y=x}$ is also a reflection in the line y=x

Correct answer A

answer 19

Left to right crossing counts for +1
Right to left crossing counts for -1
This leads to the following winding numbers:



Different explanation:
If you walk along all edges of the polygon, then you walk around area I in counter clockwise direction exactly 1 time --> winding number +1
If you walk along all edges of the polygon, then you walk around area II in clockwise direction exactly 2 times --> winding number -2

Correct answer A

answer 20

Notation: $d(P,V)$ is the signed distance of a point to a plane (positive if the point is at one side of the plane and negative if the point is at the other side of the plane)

$d(P,V) = \dfrac{\overline{p} \cdot \overline{n} + d}{|\overline{n}|}$ and $d(Q,V) = \dfrac{\overline{q} \cdot \overline{n} + d}{|\overline{n}|}$

Therefore:

"distance of P to V is equal to distance of Q to P" $\Leftrightarrow d(P,V) = \pm d(Q,V) \Leftrightarrow \overline{p} \cdot \overline{n} + d = \pm(\overline{q} \cdot \overline{n} + d)$

So, the absolute values $|\overline{p} \cdot \overline{n} + d|$ and $|\overline{q} \cdot \overline{n} + d|$ must be equal.

$\overline{p} \cdot \overline{n} + d = 3 \cdot 2 - 2 \cdot 5 + 1 \cdot -3 = 6 - 10 - 3 - 3 = -10$

In answer a) $\overline{q} \cdot \overline{n} + d = 3 \cdot 0 - 2 \cdot 5 + 1 \cdot 2 - 3 = -11$

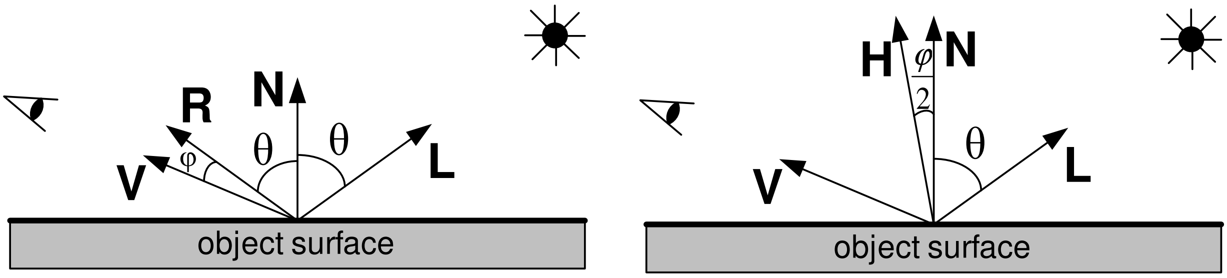
In answer b) $\overline{q} \cdot \overline{n} + d = 3 \cdot 3 - 2 \cdot 1 + 1 \cdot 5 - 3 = 9$

In answer c) $\overline{q} \cdot \overline{n} + d = 3 \cdot 6 - 2 \cdot 3 + 1 \cdot 1 - 3 = 10$

In answer d) $\overline{q} \cdot \overline{n} + d = 3 \cdot 8 - 2 \cdot 6 + 1 \cdot 2 - 3 = 11$

Correct answer C

answer 21



The specular reflection component (see figures above) is the only component that depends on vector V. If the vector R of perfect specular reflection is almost the same as V, then we see a highlight on the object surface. So, the specular reflection component depends on the angle φ between R and V. The angle between H and N, with H = (L+V) / |L+V|, is $\frac{\varphi}{2}$. This angle can also be used in the specular reflection component (instead of the angle between R and V).

The diffuse reflection component depends only on the angle θ between N and L, but not on the angle between R and V (or H and N).

Correct answer B

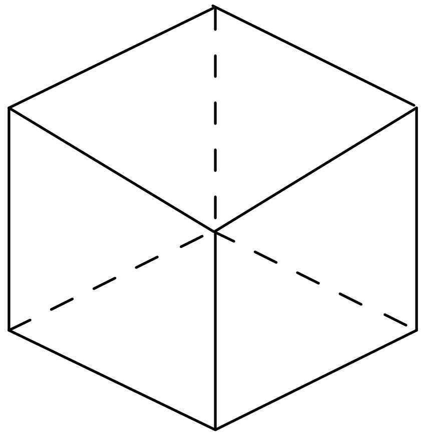
answer 22

Phong shading interpolates normal vectors and applies the light reflection model to every pixel inside a projected polygon. Gouraud shading applies the light reflection model to every vertex of a polygon network and interpolates colors.

Correct answer A

answer 23

The image of the cube is shown in the figure below.



It is not a perspective projection because parallel lines in the model (not parallel to the projection plane) are parallel in the image, instead of converging to a vanishing point.
Edges of the cube in all three main directions of the model have equal length. The cube is viewed along the main diagonal. An image like this is called an isometric projection. It is a special case of an orthographic (parallel) projection.

Correct answer A

answer 24

In ray tracing the first intersection point of the ray with an object surface is determined. For this point on the object surface a light reflection model is used in order to decide which color must be assigned to the pixel. --> answer D
The other three steps in the 3D polygon rendering pipeline (answer A, B and C) are all integrated in the process of shooting rays and intersecting these rays with the objects in the scene.

Correct answer D

answer 25

For a perspective mapping with matrix M a point (u, v) in texture space is mapped on a point (x, y) in screen space in the following way:

$$M\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} x' \\ y' \\ w \end{pmatrix} \qquad x = \frac{x'}{w} \qquad y = \frac{y'}{w}$$

If, instead of M we use λM as the matrix for a perspective mapping we get

$$\lambda M\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} \lambda x' \\ \lambda y' \\ \lambda w \end{pmatrix} \qquad x = \frac{\lambda x'}{\lambda w} = \frac{x'}{w} \qquad y = \frac{\lambda y'}{\lambda w} = \frac{y'}{w}$$

So, for both mappings (u, v) is mapped on the same point (x, y).
λM and M represent the same perspective projection.

Correct answer A

answer 26

The viewing direction is $\bar{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$; the normal vector on polygon $ax + by + cz + d = 0$ is $\bar{n} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$.

A polygon is a back face if and only if $\bar{v} \cdot \bar{n} \geq 0$, so if and only if $v_x a + v_y b + v_z c \geq 0$.

Correct answer B

answer 27

A line segment can be rejected (directly) from the line endpoint codes cA and cB if and only if cA AND cB <> 0000. So if and only if there is a 1 on corresponding bit positions in both cA and cB.

Only in situation D (with cA = 0101 and cB = 1001) there is a 1 on corresponding bit positions. The rightmost bit is 1 in both codes in this situation.

Correct answer D

answer 28

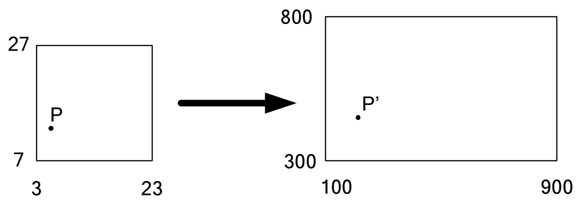
During descending the scene graph (from a parent node to a child node) the following actions are necessary in order to get the correct transformation matrix for the child object and also to be able to get back the correct transformation matrix when returning to the parent node in the scene graph:

- Save the current model view matrix M on the matrix stack --> **glPushMatrix();** The transformation matrix for the parent node is saved.
- Post multiply the current model view matrix with the transformation of the child relative to the parent. The transformation of the child relative to the parent is a translation --> **glTranslate(tx, ty, tz);** The model view matrix is now M . T(tx, ty, tz).

Later, when ascending from the child node to the parent node, the matrix M is popped from the stack.

Correct answer C

answer 29



$$\frac{23 - x_P}{23 - 3} = \frac{900 - x_{P'}}{900 - 100}$$

$$\frac{23 - 6}{23 - 3} = \frac{900 - x_{P'}}{900 - 100}$$

$$900 - x_{P'} = \frac{17}{20} \cdot 800 = 680$$

$$x_{P'} = 900 - 680 = 220$$

$$\frac{27 - y_P}{27 - 7} = \frac{800 - y_{P'}}{800 - 300}$$

$$\frac{27 - 13}{27 - 7} = \frac{800 - y_{P'}}{800 - 300}$$

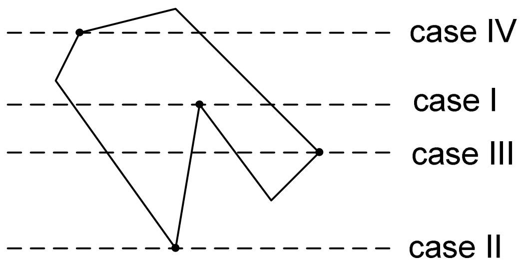
$$800 - y_{P'} = \frac{14}{20} \cdot 500 = 350$$

$$y_{P'} = 800 - 350 = 450$$

Correct answer D

answer 30

Look at the following example polygon



In **case I** both on the left and on the right side of the point we are inside the polygon. So, **two intersections** must be found.
In **case II** both on the left and on the right side of the point we are outside the polygon. So, **two intersections** must be found.
In case III on the left side of the point we are inside the polygon and on the right side we are outside the polygon. So, one intersection point must be found.
In case IV on the left side of the point we are outside the polygon and on the right side we are inside. So, one intersection point must be found.

Correct answer B